

Stronger constraints on non-Newtonian gravity from the Casimir effect

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Abstract. We review new constraints on the Yukawa-type corrections to Newtonian gravity obtained recently from gravitational experiments and from the measurements of the Casimir force. Special attention is paid to the constraints following from the most precise dynamic determination of the Casimir pressure between the two parallel plates by means of a micromechanical torsional oscillator. The possibility of setting limits on the predictions of chameleon field theories using the results of gravitational experiments and Casimir force measurements is discussed.

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1. Introduction

During the last ten years hypothetical long-range interactions coexisting with Newtonian gravity have received much attention. There are serious reasons why the existence of such interactions is very probable. Many extensions of the standard model predict light elementary particles, such as axions, scalar axions, dilatons, graviphotons, etc. The exchange of such particles between two atoms with masses M_1 and M_2 at a separation r results in an attractive or repulsive force described by the effective Yukawa-type potential which is added to the usual gravitational potential [1]

$$V_{\text{Yu}}(r) = -\frac{GM_1M_2}{r} \left(1 + \alpha e^{-r/\lambda}\right). \quad (1)$$

Here, G is the gravitational constant, α is the interaction constant of a hypothetical interaction relative to gravity, and λ is the interaction range ($\lambda = m^{-1}$ where m is the mass of a hypothetical particle). Exchange of massless particles (neutrinos or arions,

for instance) leads to the power-type corrections to Newtonian gravity with different powers [2, 3]

$$V_l(r) = -\frac{GM_1M_2}{r} \left[1 + \Lambda_l \left(\frac{r_0}{r} \right)^{l-1} \right], \quad (2)$$

where $l = 1, 2, 3, \dots$, Λ_l is the interaction constant and the arbitrary parameter $r_0 = 10^{-15}$ m is introduced for preserving the proper dimensionality of the potential.

Another theoretical scheme that predicts corrections to Newton's gravitational law is extra-dimensional physics with low compactification energy $M_{\text{Pl}}^{(N)} = 1/G_{4+n}^{1/(2+n)} \sim 1$ TeV, where G_{4+n} is the gravitational constant in $N = (4+n)$ -dimensional space-time and n is the number of extra spatial dimensions. This energy should be compared with the usual Planck energy $M_{\text{Pl}} = 1/\sqrt{G} \sim 10^{19}$ GeV. The size of the compact extra dimensions is given by [4, 5]

$$R_n \sim \frac{1}{M_{\text{Pl}}^{(N)}} \left(\frac{M_{\text{Pl}}}{M_{\text{Pl}}^{(N)}} \right)^{2/n} \sim 10^{\frac{32}{n}-17} \text{ cm}. \quad (3)$$

Under the condition that $r \gg R_n$, low energy compactification schemes predict Yukawa-type corrections to Newtonian gravity, as in (1), with $\lambda \sim R_n$ [6, 7]. For $n = 1$ it follows that $R_1 \sim 10^{15}$ cm which is excluded by gravity tests in the solar system [1]. However, for $n = 2$ and 3 the sizes of predicted extra dimensions are $R_2 \sim 1$ mm and $R_3 \sim 5$ nm, i.e., the very ones presently tested in the laboratory experiments of Cavendish- and Eötvös-type and in the measurements of the Casimir force.

Another proposed scheme deals with noncompact but warped extra dimensions [8] and this leads to power-type correction to Newtonian gravity, as in (2), with $l = 3$.

Recently one more extension of the standard model, the so-called *chameleon field theory*, became very popular. As with many other extensions of the standard model, this theory introduces one or more scalar fields. A specific feature of these fields is that their masses depend on the local background matter density and they can couple directly to matter with gravitational strength [9, 10]. The chameleon scalar field, if it really exists in nature, leads to an additional chameleon force acting between two nearby macrobodies. The functional dependence of this force on the separation distance is rather complicated and it depends on the specific form of the potential of the chameleon field. Typically the chameleon force behaves as an inverse power of distance between the two macrobodies but other asymptotic regimes are also possible [11, 12].

All of the above predictions made in physics beyond the standard model can be tested using gravitational experiments and measurements of the Casimir force. In this paper we briefly review the progress achieved in the strengthening of constraints on non-Newtonian gravity during the two years passed after the QFEXT05 conference in Barcelona. In Section 2 new constraints obtained from precise gravitational measurements are presented. Section 3 is devoted to the constraints following from the most precise determination of the Casimir pressure between the two parallel plates using a micromechanical torsional oscillator. Section 4 contains our conclusions and prospects. We use units with $c = \hbar = 1$.

2. Constraints following from gravitational experiments

Gravitational experiments of the Eötvös- and Cavendish-type have a long history. They have been considered as the most precise physical experiments over many years. Eötvös-type experiments measure limits on the relative difference in accelerations imparted by the Earth, Sun or some laboratory attractor to various substances of the same mass. In Cavendish-type experiments, limits on the deviations from the force-distance dependence of $1/r^2$ in the Newton gravitational law are measured. The results of both types of experiments can be used to constrain the interaction constants (λ, α) and Λ_l in the interaction potentials (1), (2) [13]. In figure 1 we present the strongest constraints obtained from gravitational experiments on the parameters of a Yukawa-type hypothetical interaction (λ, α) . Lines 1, 2 and 3 are obtained from the experiments of papers [14, 15, 16], respectively. Permitted regions on (λ, α) -plane lie beneath the lines.

During the last two years, constraints on the parameters of Yukawa-type interactions were strengthened in two new important gravitational experiments [17, 18]. In [17] a micromechanical cantilever was used as the force sensor, and its displacement was measured interferometrically to find constraints on the Yukawa-type deviations from Newtonian gravity. The results of this experiment are shown as line 5 in figure 1. The largest improvement over previous results obtained in [19, 20] is by almost a factor of 10 at $\lambda \approx 20 \mu\text{m}$. The experiment [18] sets stronger constraints on deviations from Newton's inverse-square law using a torsion-pendulum detector suspended above an attractor that was rotated with a uniform angular velocity. The resulting constraints are shown as line 4 in figure 1. The previously known constraints in this region found in [17, 19, 21] are improved by a factor of up to 100 by the results of this experiment (line 6 shows constraints [22] following from the measurement of the Casimir force using a torsion pendulum [23]). This shows that gravitational experiments have considerable potential in further strengthening the constraints on Yukawa-type corrections to Newtonian

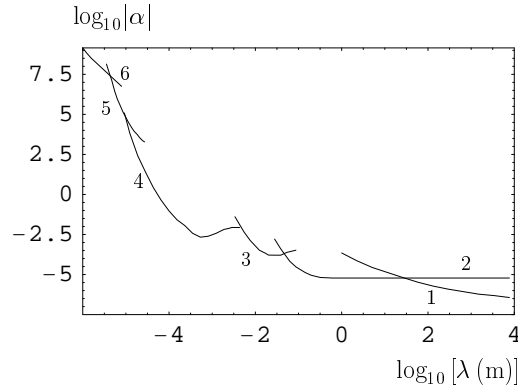


Figure 1. Strongest constraints on Yukawa-type corrections to Newton's gravitational law following from different gravitational experiments (lines 1–5) and the measurement of the Casimir force (line 6). Permitted regions in the (λ, α) -plane lie beneath the lines (see text for further discussion).

gravity for λ larger than a few micrometers. At the same time, constraints on the parameters Λ_i of power-type interactions have not been strengthened during the last time (see [24] for the list of the strongest constraints).

The gravitational experiments have the potential to constrain some predictions of chameleon theories. The predictions of these theories depend, however, on where the gravitational experiment is performed. If it is performed in the low-density vacuum of space, the magnitude of the chameleon force might be larger than if it is performed in the relatively high-density environment of a laboratory. According to [12], the experiment [18] could detect or rule out the existence of chameleon fields with some natural values of parameters, provided it is designed to do so. In particular, the role of electrostatic forces should be eliminated without using a metallic sheet between the attractor and pendulum. Such a sheet is used presently, but it plays the crucial role when testing for chameleon fields.

3. Constraints on the Yukawa interaction from Casimir force measurements

Measurements of the Casimir force are now generally recognized as the source of constraints on Yukawa-type corrections to Newtonian gravity. During the last few years significant progress has been made in increasing the experimental precision and in the comparison of the measurement data with theory at a given confidence level [25, 26]. This has permitted us to obtain constraints of the same reliability as those following from the gravitational experiments. Typically measurements of the Casimir force allow one to obtain constraints on hypothetical interactions with a shorter interaction range than gravitational experiments. Thus, both types of experiments play a supplementary role in constraining the hypothetical interactions of Yukawa-type.

The basic idea on how the Casimir force measurements can be used for constraining hypothetical long-range interactions is the following. The hypothetical interaction of Yukawa-type (1) leads to some additional force in the experimental configuration where the Casimir force is measured. This additional force depends on unknown parameters α and λ . If the measurement data for the Casimir force are consistent with respective theory within some confidence interval, the hypothetical force must be sufficiently small. This imposes constraints on α and λ .

Here we present constraints on Yukawa-type corrections to Newton's gravitational law following from recent dynamic determinations of the Casimir pressure between the two gold coated parallel plates by means of a micromechanical torsional oscillator [27, 28]. In this experiment a large sphere is oscillating above a plate with the natural frequency of the oscillator and the frequency shift due to the Casimir force is measured. By means of the proximity force approximation, the frequency shift is recalculated into the equivalent Casimir pressure between two plates. The experiment under discussion is the first measurement of the Casimir force of metrological quality in the sense that the stochastic experimental error is much smaller than the systematic error. As a result, it is the systematic error alone that determines the total experimental error over

the entire measurement range. The total experimental error of the Casimir pressure measurements determined at a 95% confidence level varies from 0.19% of the measured pressure at a separation $a = 162$ nm, to 0.9% at $a = 400$ nm, and to 9.0% at $a = 746$ nm. The description of the experimental setup, the measurement procedure, and of the comparison of data with theory can be found in [27, 28].

Constraints on the Yukawa-type hypothetical interaction are obtained from the measure of agreement between the experimental data and theory. This can be quantified as a 95% confidence band $[-\tilde{\Xi}(a), \tilde{\Xi}(a)]$ containing no less than 95% of all differences $P^{\text{th}}(a) - \bar{P}^{\text{exp}}(a)$ in the measurement range from 180 to 746 nm, where $P^{\text{th}}(a)$ is the calculated value of the Casimir pressure at a separation a and $\bar{P}^{\text{exp}}(a)$ is the mean measured value at the same separation. The function $\tilde{\Xi}(a)$ is determined by both the experimental errors discussed above and the theoretical errors in the calculation of the Casimir pressure. In [28] $\tilde{\Xi}(a)$ was determined in a conservative way, such that the confidence band $[-\tilde{\Xi}(a), \tilde{\Xi}(a)]$ includes not only 95%, but 100% of differences between the theoretical and mean experimental Casimir pressures (note that data from the shortest separations between 162 and 180 nm were not used for obtaining constraints). For example, at typical separations $a = 180, 200, 250, 300, 350, 400$ and 450 nm, the half-widths of the confidence band are equal to $\tilde{\Xi}(a) = 4.80, 3.30, 1.52, 0.84, 0.57, 0.45$, and 0.40 mPa, respectively. From this, the magnitude of the hypothetical pressure can be constrained from the inequality

$$|P^{\text{hyp}}(a)| \leq \tilde{\Xi}(a). \quad (4)$$

The constraints obtained from (4) are characterized by the same confidence as $\tilde{\Xi}(a)$, i.e., by the 95% confidence level.

The hypothetical pressure resulting from the potential (1) can be obtained by the integration of (1) over the volumes of the plates, and subsequent negative differentiation with respect to a . In so doing the contribution from the gravitational interaction [the first term in (1)] can be neglected [29, 30]. In this dynamic experiment one plate is effective and has the same layer structure as a large oscillating sphere of radius R . Thus, it is made of sapphire of density $\rho_s = 4.1 \text{ g/cm}^3$ coated with a layer of Cr of density $\rho_c = 7.14 \text{ g/cm}^3$ and thickness $\Delta_c = 10 \text{ nm}$, and then with an external layer of gold of thickness $\Delta_g^{(s)} = 180 \text{ nm}$ and density $\rho_g = 19.28 \text{ g/cm}^3$. The other (real) plate is made of Si of thickness $L = 3.5 \mu\text{m}$ and density $\rho_{Si} = 2.33 \text{ g/cm}^3$. It was first coated with a layer of Cr of $\Delta_c = 10 \text{ nm}$ thickness and then with a layer of gold of $\Delta_g^{(p)} = 210 \text{ nm}$ thickness. Note that both sapphire and Si can be considered as infinitely thick. Under the conditions $a, \lambda \ll R$, the equivalent Yukawa pressure between the two parallel plates with the above layer structure is given by [22, 31]

$$\begin{aligned} P^{\text{hyp}}(a) = & -2\pi G\alpha\lambda^2 e^{-a/\lambda} \\ & \times \left[\rho_g - (\rho_g - \rho_c)e^{-\Delta_g^{(s)}/\lambda} - (\rho_c - \rho_s)e^{-(\Delta_g^{(s)} + \Delta_c)/\lambda} \right] \\ & \times \left[\rho_g - (\rho_g - \rho_c)e^{-\Delta_g^{(p)}/\lambda} - (\rho_c - \rho_{Si})e^{-(\Delta_g^{(p)} + \Delta_c)/\lambda} \right]. \end{aligned} \quad (5)$$

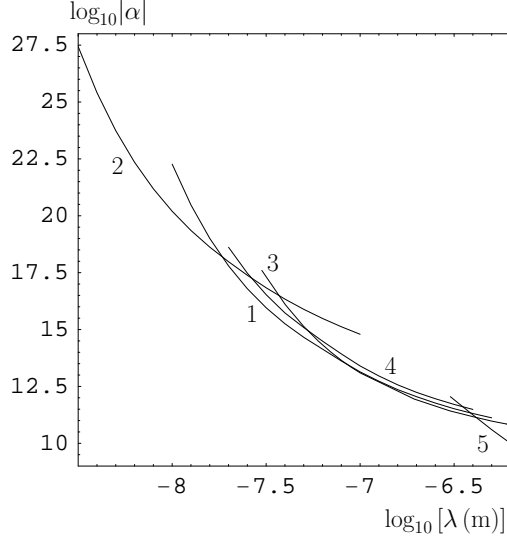


Figure 2. Constraints on Yukawa-type corrections to Newton's gravitational law following from different measurements of the Casimir force (see text for further discussion). Permitted regions on (λ, α) -plane lie beneath lines 1–5.

We have substituted (5) in (4) and found constraints on the parameters of Yukawa interaction λ , α at different separations a . The strongest constraints are shown in figure 2 by line 1. For different λ , the strongest constraints are obtained at different separations a . As an example, for $10 \text{ nm} < \lambda < 56 \text{ nm}$, the comparison of experiment with theory at a separation of $a = 180 \text{ nm}$ leads to the strongest constraints. For illustration, constraints from earlier experiments are also shown in figure 2. Line 2 follows from the short-separation measurement of the Casimir force between a sphere and a plate using an atomic force microscope [32, 33]. Note that for the first time the constraints following from this experiment were obtained in [29] at an undetermined confidence level. Here, line 2 is recalculated at a 95% confidence level using the improved procedure for the comparison of the Casimir force measurements with theory, as described in [25, 26]. Line 3 was obtained in [34] using the isoelectronic technique. Line 4 follows from the previous experiment of the dynamic determination of the Casimir pressure by means of a micromechanical torsional oscillator [25]. Line 5 was obtained [22] from the measurement of the Casimir force using a torsion pendulum [23]. It is, in fact, a continuation of the line labeled 6 in figure 1.

As is seen in figure 2, the constraints represented by line 1 are the strongest ones within the interaction range from 20 to 86 nm. The largest improvement of previously obtained results is by a factor of 4.4 at $a = 26 \text{ nm}$.

It is of interest to consider constraints on the predictions of chameleon field theories which follow from the measurements of the Casimir force. The typical potential of the chameleon field ϕ can be chosen in the form [11]

$$V(\phi) = \Gamma_0^4 \left(1 + \frac{\Gamma^n}{\phi^n} \right), \quad (6)$$

where n can be both positive and negative, and Γ_0, Γ are some constants. To fit data for the acceleration of the Universe, one requires $\Gamma_0 \approx 2.4 \times 10^{-3} \text{ eV}$. The hypothetical pressure P_ϕ arising between the two parallel plates in chameleon theories with potential (6) and $\Gamma = \Gamma_0$ was calculated in [11]. It was shown that for $n > 0$ the most precise experimental results of [27, 28] do not impose any constraints on predictions of chameleon theories. The current limits in figure 2 should be strengthened by at least two orders of magnitude in order for constraints on chameleon theories with $n > 0$ to be obtained. At the same time the experimental data of [27, 28] rule out the chameleon theories with $n = -4$ and $n = -6$ [11]. Future Casimir force measurements at large separations can be used to obtain more stringent constraints on the predictions of chameleon field theories.

4. Conclusions and discussion

As was discussed above, during the last two years new important gravitational experiments and Casimir force measurements have been performed which lead to stronger constraints on Yukawa-type corrections to Newtonian gravity. The stronger constraints obtained from the gravitational measurements are related to the interaction range from about $4 \mu\text{m}$ to $4000 \mu\text{m}$. Constraints strengthened from the measurement of the Casimir pressure between two parallel plates are related to shorter interaction scales from 20 to 86 nm. Thus, both experimental approaches used to strengthen constraints on non-Newtonian gravity are complementary.

One important innovation introduced in the measurements of the Casimir force during the last years is the increased experimental precision that permitted us to obtain data of metrological quality, where stochastic errors are much below the systematic errors. Another innovation is the use of rigorous statistical procedures for data processing and for the comparison of experiment with theory, which allowed us to obtain constraints at a fixed high confidence level. Taken together, these innovations significantly increased the reliability of the resulting constraints on non-Newtonian gravity, bringing them closer to the previously achieved high reliability constraints following from the gravitational experiments.

An interesting new direction, which came into being recently, is the application of Casimir force measurements to obtain constraints on the predictions of chameleon field theories. First results in this direction have been already obtained (see above). New experiments planned for the near future promise to provide much more information on this subject, especially if the chameleon theories become more certain than they presently are. In this respect a more precise laboratory technique for probing small forces in submicrometer range (see, e.g., [35]) is of high promise.

All this permits us to conclude that relatively inexpensive laboratory measurements of the Casimir force continue to have great potential to obtain new information on elementary particles and fundamental interactions.

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